The Chain Rule

Chain Rule: If $y = f(u)$ is differentiable function of u and $u = g(x)$ is a differentiable function of x, then $y = f(g(x))$ is a differentiable function and

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

or, equivalently,

$$\frac{d}{dx}[f(g(x))] = f'\big(g(x)\big)g'(x)$$

EX:

$y = f(g(x))$	$u = g(x)$	$y = f(u)$
$y = \frac{1}{\sqrt{x+1}}$		
$y = \tan(\pi x + 1)$		
$y = \sqrt{3x^2 - x + 1}$		

EX: Find y' for $(3x^2 + 1)^4$

General Power Rule: If $y = [u(x)]^n$, where u is a differentiable function of x and n is a rational number, then

$$\frac{dy}{dx} = n[u(x)]^{n-1}\frac{du}{dx}$$

Or, equivalently,

$$\frac{d}{dx}[u^n] = nu^{n-1}u'$$

EX: Find the derivative: $f(x) = \frac{2}{(1-x^2)^3}$

EX: $y = sin4x^2$

EX: $f(x) = x^2\sqrt[3]{x^2+5}$

EX: $h(t) = \left(\frac{t^2}{t^{3+2}}\right)^2$

EX: $y = e^{-x^2}$